MULTIPLE STEP FINANCIAL TIME SERIES PREDICTION WITH
PORTFOLIO OPTIMIZATION

by

David Hugo Diggs

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Abstract

The Time Series Data Mining framework developed by Povinelli is extended to perform weekly multiple time-step prediction and adapted to perform weekly stock selection from a broader market. The stock selections are combined into weekly portfolios, and techniques from Modern Portfolio Theory and the Capital Asset Pricing Model are adapted to optimize the portfolios. The contribution of this work is the combination of stock selection and portfolio optimization to develop a temporal data mining based stock trading strategy. Results show that investors can increase overall wealth, obtain optimal weekly portfolios that maximize return for a given level of portfolio risk, overcome trading costs associated with trading on a weekly basis, and outperform the market over a given time range.
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Chapter 1 Introduction

1.1 Motivation

The financial markets are perennially attractive to researchers from a wide range of fields [1, 2]. This attraction is due to the lure of easy money, if only a method to perfectly predict the dynamics of the market could be discovered. From a more scientific stance, the financial markets are interesting because of their incredible complexity and time varying dynamics. The interests of millions of investors are represented through the rise and fall of stocks prices and the companies they represent.

The dynamics of the stock market have been modeled in many ways. Box and Jenkins showed that the ARIMA model was a good first approximation [3, 4]. This model can be understood as a random walk process. Further financial research has produced more robust versions of the random walk model including the efficient market hypothesis [5]. All of these models of the market assume that all information is represented in the market immediately and that any attempt to profit through arbitrage will fail, because the stocks are correctly valued.

However, despite this theory, there is a never-ending attempt by technicians to model the stock market dynamics to achieve superior returns [6, 7]. Examples of statistical trading strategies include chart analysis, momentum or swing trading, and trend trading [1, 6-8]. Each of these strategies attempts to outperform market benchmarks by providing above market returns without significantly increasing risk.

Recently, stock market research has become more attractive because of the increased access to financial data and the ability to invest independently. Websites such as http://finance.yahoo.com, http://esignal.com, and http://moneycentral.msn.com give
investors access to current and reliable financial information along with overall market conditions for investing. Online trading sites such as TDWaterhouse.com, Etrade.com, and Ameritrade.com, have allowed small investors to easily set up and manage their own investments. This combination of reliable financial data access and easy online trading is important in developing and testing an investment strategy.

The work presented in this thesis is in the nature of a technical approach. We attempt to identify hidden patterns in the market data that are predictive of increases in a stock price. The unique nature of this work is the combination of dynamical systems theory with portfolio optimization techniques and the study of this approach across different prediction horizons and market conditions [9].

1.2 Problem Statement

The goal of this research is to create a profitable trading strategy that overcomes transaction cost and outperforms the overall market returns. The proposed trading strategy combines stock selection, asset allocation, and risk management techniques. Stock selection is the process of identifying assets that have desired characteristics, and asset allocation is the process of weighting individual assets to build a portfolio. Risk management is the process of identifying and minimizing the impact of uncertain events. Asset allocation and risk management can be used to reduce risk by diversifying a portfolio. Portfolio optimization is the integration of asset allocation and risk management to create portfolios that meet specific risk and return criteria.

In this research, stock selection is accomplished using a nonlinear time series prediction approach [3]. The approach seeks to discover hidden structures in reconstructed phase spaces of the stock price time series to make predictions on future
stock price movements. The details of reconstructed phase spaces and the data mining approach for stock selection are found in Chapter 2.

Once stock selection is completed, optimal portfolios are constructed using techniques based on Modern Portfolio Theory and the Capital Asset Pricing Model [2, 7]. Modern Portfolio Theory, developed by Harry Markowitz, makes the assumption that investors differ only in their expectations of return required for a particular investment and risk tolerance. Modern Portfolio Theory provides the techniques to create a set of portfolios that are optimal in the sense that they maximize portfolio return for a given level of portfolio risk [2, 5, 9]. The Capital Asset Model extends Modern Portfolio Theory by determining a method for selecting a specific optimal portfolio from a set of optimal portfolios.

This research contributes a trading strategy that employs a temporal data mining approach to stock selection combined with portfolio optimization. The trading strategy trades periodic weekly portfolios, by buying the entire portfolio at the beginning of the period and selling it at the end of the period.

1.3 Thesis Outline

This thesis consists of five chapters. Chapter 2 reviews the Time Series Data Mining Method (TSDM), the stock selection approach used here, and traditional portfolio optimization techniques. Chapter 3 describes the problem-specific methods used in stock selection and portfolio optimization. It also presents the extensions and adaptations of the TSDM method along with the adapted portfolio optimization techniques used to develop the proposed trading strategy. Chapter 4 evaluates the proposed methods on historical data. This chapter details the stock market data sets, performance calculations, and
experimental results. Chapter 5 discusses the research results, conclusions, and suggestions for future directions.
Chapter 2 Background

The Time Series Data Mining (TSDM) framework transforms market time series into reconstructed phase spaces (RPSs) and searches these phase spaces for temporal structures predictive of the greatest changes in the market time series [3]. This framework combined with portfolio optimization, which involves modifying the weights of the assets in a portfolio to achieve a specific investor goal or set of goals, is used to formulate a portfolio trading strategy. The portfolio optimization techniques used here are based on Modern Portfolio Theory (MPT) and the Capital Asset Pricing Model (CAPM) [1, 5]. The chapter presents an overview of the components used in developing the proposed trading strategy.

2.1 Temporal Data Mining Overview

A time series is an ordered sequence of real-valued elements denoted by

\[ x = x_n, \quad n = 1, \ldots, N, \] (2.1)

where \( n \) is the current time index, and \( N \) is the number of observations. Time series appear in many forms in a variety of fields. Domains such as medicine, speech, and finance have applications that involve the study of temporal data [10-14]. This thesis applies temporal data mining techniques in the area of financial time series prediction.

Temporal data mining is a sub-field of data mining that focuses primarily on discovering relationships between sequences of real valued time series events. Techniques common to data mining and temporal data mining are association rule learning, classification, clustering, and prediction [15-18]. The main difference between temporal data mining and data mining in general is in how the data is represented. Often time series signals are noisy, non-linear, and chaotic, making patterns and data
relationships hard to detect [19]. Linear and nonlinear time series transforms such as linear filters and time series embedding techniques are used to modify the representation of time series data without losing valuable information about the time series. Specific time series transformation techniques such as the Discrete Fourier Transform, which transforms a signal from the time domain into the frequency domain, and the Discrete Wavelet Transform, which translates a time series into the time-frequency domain, have been used to represent data in formats suitable for data mining tasks [20].

The TSDM approach has its foundation in temporal data mining using techniques from machine learning, artificial intelligence, and genetic algorithms. The approach uses a time-delay embedding technique called phase space reconstruction that creates a time-lagged version of the original signal [3, 10]. The next section presents the concept and theoretical definition of a reconstructed phase space.

2.2 Reconstructed Phase Space

This section discusses the definition of a reconstructed phase space and the theoretical justification for using the technique in this thesis. The reconstructed phase space is a time-delay embedding of an original time series and has been shown to capture nonlinear information found in complex dynamical systems that have many dimensions [3, 10, 21]. This technique creates a time-lagged version of a signal used to discover hidden patterns normally not detected in a linear space. This approach provides the basis for the data mining-based stock selection process presented later in this thesis.

A reconstructed phase space (RPS) is a $d$-dimensional metric space in which a time series is unfolded. Takens proved that if the dimension of the embedding space is large enough, then the RPS is topologically equivalent to the original state space that
generated the time series [20, 22, 23]. The RPS can be formed using a time delay embedding process, which performs a homeomorphic mapping from one topological space to another. The embedding process creates the RPS signal, which is a time-delayed version of the original time series signal [19, 20, 23]. It maps a set of $d$ time series observations taken from a time series $x$ on to

$$x_n = \begin{bmatrix} x_{n-(d-1)\tau} & \cdots & x_{n-\tau} & x_n \end{bmatrix}$$

which is a vector or point in the phase space. Together the phase space points form a trajectory matrix

$$X = \begin{bmatrix} x_{1+(d-1)\tau} \\ x_{2+(d-1)\tau} \\ \vdots \\ x_{N-(d-1)\tau} \end{bmatrix} = \begin{bmatrix} x_1 & x_{1+\tau} & \cdots & x_{1+(d-1)\tau} \\ x_2 & x_{2+\tau} & \cdots & x_{2+(d-1)\tau} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-(d-1)\tau} & x_{N-d\tau} & \cdots & x_N \end{bmatrix}_{(N-(d-1)\tau) \times d}$$

where $d$ is the embedding dimension, $\tau$ is the time-lag, and $x_n$ is the signal value at time index $n$. Figure 2.2 shows an example of a RPS (a plot of the trajectory matrix) from a randomly generated time series shown in Figure 2.1. The original time series is time-delay embedded with a dimension of two to create the RPS. Equation 2.4 shows the first five points in the trajectory matrix from Figure 2.2.

$$X = \begin{bmatrix} 1 & 6 \\ 6 & 9 \\ 9 & 3 \\ 3 & 8 \\ 8 & 1 \end{bmatrix}$$

Takens proved that if an embedding of a time series is performed correctly, then the dynamics of the RPS are topologically equivalent to the original state space and the RPS contains the same topological information as the original state space of system [21].
Therefore, characterizations and predictions based on the RPS are considered valid and similar to those made if the original state space were available.
2.3 Genetic Algorithm

The Time Series Data mining method uses a simple genetic algorithm as an optimization method to discover predictive hidden patterns, with high fitness values, in the reconstructed phase space. A genetic algorithm is a method of problem solving and global optimization that uses computational models of evolutionary processes as elements in design and implementation [24]. Genetic algorithms incorporate aspects of natural selection to maintain a population of structures that evolves according to rules of selection, recombination, mutation, and survival of the fittest. The fitness or performance of each individual in the population determines which individuals are more likely to be selected for reproduction, while recombination and mutation modify those individuals, yielding potentially superior ones. This process leads to fitter populations corresponding to better solutions to various problems. Genetic algorithms have been shown useful in finding optimal solutions in non-linear functions [24].

The main concepts of a binary genetic algorithm are fitness, objective function, chromosome, population, and generation [24]. A chromosome is a binary encoding of the independent variables of the objective function. The fitness of a chromosome is the application of the objective function to a decoded chromosome. A population is a set of chromosomes. A generation is one iteration of the genetic algorithm, which is comprised of the application of a set of operators to the population. The most frequently used operators in genetic algorithms are selection, crossover, mutation, and reinsertion [24].

An objective function defines a rule for the search space where the optimizer is to be found. A simple example of an objective function might be:

\[ f(x) = x^2 + x + 100. \] (2.5)
An example of a small population for a particular generation is shown in Table 2.1 with associated fitness values and chromosome lengths of eight.

<table>
<thead>
<tr>
<th>Chromosome</th>
<th>$x$</th>
<th>$f(x)$, fitness</th>
</tr>
</thead>
<tbody>
<tr>
<td>00000000</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>01111111</td>
<td>127</td>
<td>16356</td>
</tr>
<tr>
<td>11111100</td>
<td>-4</td>
<td>112</td>
</tr>
</tbody>
</table>

Table 2.1 Chromosome Example

The first operator in a iteration of a genetic algorithm is typically selection. It is the process of choosing chromosomes from a population based on each chromosome’s fitness. The type of selection used in this work is roulette wheel selection in which a chromosome is given a section of the roulette wheel based on the size of its fitness value. The wheel is spun once, and the winning chromosome is selected for further permutations.

The next typical operator is crossover, which is the process of combining chromosomes in a manner similar to sexual reproduction. The crossover operator combines segments from the encoded format of each parent to create offspring chromosomes shown in Figure 2.3. Crossover can be accomplished using either a fixed or a random crossover locus.
An example of the crossover process is again showed in Table 2.2 with example chromosomes.

<table>
<thead>
<tr>
<th>Mating pair</th>
<th>Parent 1</th>
<th>Parent 2</th>
<th>Offspring 1</th>
<th>Offspring 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1111↑1100</td>
<td>0000↑0000</td>
<td>0000↑1100</td>
<td>1111↑0000</td>
</tr>
<tr>
<td>2</td>
<td>0000↑0000</td>
<td>1010↑1111</td>
<td>0000↑1111</td>
<td>1010↑0000</td>
</tr>
</tbody>
</table>

Table 2.2 Crossover Process Example

The mutation operator randomly changes the bits of the chromosomes as shown in Table 2.3. The mutation operator is usually set at a specific mutation rate is used to control an aspect of population evolution and periodically randomize the population to avoid local minimums and maximums.

<table>
<thead>
<tr>
<th>Pre-mutation</th>
<th>Post-mutation</th>
</tr>
</thead>
<tbody>
<tr>
<td>00001111</td>
<td>00011111</td>
</tr>
<tr>
<td>01010011</td>
<td>01011011</td>
</tr>
</tbody>
</table>

Table 2.3 Mutation Example

Reinsertion is the process of selecting only a small percentage of chromosomes to bypass the operations of selection, crossover, and mutation. This technique allows the
individuals with the highest fitness to pass directly to the next generation without being modified and ensures that elite individuals are not lost due to the stochastic nature of selection and crossover. A genetic algorithm uses these steps, shown in Figure 2.4, to find objective function optimizers.

![Figure 2.4 Genetic Algorithm Process](image)

The simple genetic algorithm described above performs a search in the reconstructed phase space, generating subsequent chromosome populations until a stopping criterion is met and a highly predictive temporal structure is found. The steps in the simple genetic algorithm process are:
• Random population initialization

• Calculate fitness

• While fitness value have not converged
  o Selection
  o Crossover
  o Mutation
  o Reinsertion

2.4 Time Series Data Mining

Time Series Data Mining employs a time-delay embedding process that embeds a
time series into a reconstructed phase space (RPS), shown in Figures 2.1 and 2.2. The
RPS, discussed in Section 2.2, is topologically equivalent to the original system that
generated the time series [22, 23]. The TSDM method also uses a genetic algorithm
search, discussed in Section 2.3, to discover hidden temporal structures in a time-delay
embedded signal that are characteristic and predictive of time series events, where
temporal structures are a predictive sequence of points found in time series data that
signal future outcomes and events. The temporal structures found in the time series data
are used to predict sharp movements in a time series. Originally, the TSDM technique
was applied to making one-step time series predictions such as predicting sharp increase
in daily stock price or welding droplet release times [10, 25, 26]. Here it is applied to
make predictions on a weekly basis [27].

To better explain the TSDM method, we introduce a set of concepts. The concepts
are opportunities, events, goal function, temporal pattern, temporal structure,
reconstructed phase space, augmented phase space, average event function, and ranking
function. Figure 2.5 shows how the concepts relate to the overall Time Series Data Mining method. As with machine learning approaches, the method is composed of a training stage and a testing stage. The training stage defines the prediction goal and identifies predictive structures in training signal of the embedded time series data. The testing stage uses the predictive temporal structure found during training to predict events.

![Diagram of Time Series Data Mining Method](image)

### 2.4.1 Concepts and Definitions

Time Series Data Mining method concepts are defined and explained with examples for each concept. Each concept refers to a step in the TSDM method and defines the actions taken in each step.

*Events* are defined as important occurrences in time. *Opportunities* are defined as chances to take advantage of significant events that occur over time. Events and
opportunities are discovered in time series data such as a stock price time series shown as,

\[ x = x_n, \quad n = 1, ..., N, \]  

(2.5)

which represent the price movements of a stock over a time period with length \( N \) and price \( x_n \). An important occurrence in the stock price time series is an increase in the stock price. For example, the rise in a stock price, over a given period, represents an opportunity to take action by having purchased the stock before the start of that time period. Figure 2.6 shows the daily stock time price time series for General Motors (GM) from 1/01/2004 to 2/01/2004.

![Figure 2.6 GM Stock Price Time Series 1/01/2004 –2/01/2004](image)

A prediction is defined as the expectation of the future price for a stock. Predictions are labeled to determine the value and assessment of the prediction. A \textit{goal function} \( g \),
associates a future value to predictions made at the current time index \( n \). The goal function provides a mapping between the temporal structures found and the events predicted. For example, a goal function can be the one period percent change in a stock price at time index \( n \) given as,

\[
g_n = \frac{(x_{n+1} - x_n)}{x_n}.
\]  

(2.6)

A temporal pattern \( tp \) is a hidden pattern in a time series that is characteristic and predictive of occurrences. A temporal pattern, \( tp \in \mathbb{R}^D \), is defined as a vector of length \( D \) or equivalently as a point in a \( D \)-dimensional real metric space.

A temporal structure \( TS \) is defined as the surrounding set of all points within \( \delta \) of the temporal pattern shown as,

\[
TS = \{ a \in \mathbb{R}^D : d(tp, a) \leq \delta \},
\]

(2.7)

where \( d \) is the Euclidean distance metric defining a hyper-sphere with center \( tp \) and radius \( \delta \). Figure 2.7 shows an example of a temporal structure used to predict an event with the associated prediction value.
A reconstructed phase space (RPS) is a $d$-dimensional real metric space into which the time series is embedded as shown in Section 2.2. The reconstructed phase space signal is a time-lagged version of the original time series signal [19, 20]. Figure 2.8 shows General Motors’ percent change time series reconstructed into the phase space. Table 2.4 highlights the numerical mapping of the first five points in General Motors’ percent change time series to equivalent points in the phase space with an embedding dimension of 2.
The augmented phase space is a $d+1$ dimensional space formed by extending the phase space with the additional dimension of $g_n$. The augmented phase gives visualization to the value of the temporal structures in the reconstructed phase. The

<table>
<thead>
<tr>
<th>Original Time Series $(x_t, y_t)$ Coordinates</th>
<th>Reconstructed Phase Space $(x_t, x_{t+1})$ Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>02-Jan-2003, 0.000</td>
<td>(0.000, -0.0105)</td>
</tr>
<tr>
<td>03-Jan-2003, -0.0105</td>
<td>(-0.0105, 0.0246)</td>
</tr>
<tr>
<td>06-Jan-2003, 0.0246</td>
<td>(0.0246, 0.0089)</td>
</tr>
<tr>
<td>07-Jan-2003, 0.0089</td>
<td>(0.0089, -0.0409)</td>
</tr>
<tr>
<td>08-Jan-2003, -0.0409</td>
<td>(-0.0409, 0.0338)</td>
</tr>
</tbody>
</table>

Table 2.4 Phase Space Points
augmented phase space illustrated in Figure 2.9 represents the extension from the reconstructed phase space in Figure 2.8.

![Figure 2.9 Augmented Phase Space](image)

The *average event function* $\mu_M$ represents a fitness value given to a temporal structure, $TS$. This function maps a structure onto the real line to allow the temporal structures to be ranked ordered. The average value $\mu_M$, of the points that are within a temporal structure is

$$\mu_M = \frac{1}{c(M)} \sum_{r \in M} g_{r},$$  

(2.8)

where $c(M)$ is the cardinality of $M$, the set of all points that are within a temporal structure. In contrast, the average value $\mu_{\overline{M}}$ of the points that are not within a temporal structure is denoted as

$$\mu_{\overline{M}} = \frac{1}{c(M)} \sum_{r \in \overline{M}} g_{r},$$  

(2.9)
where \( c(\tilde{M}) \) is the cardinality of \( \tilde{M} \), the set of all points not within a temporal structure.

A ranking function \( f \), shown in Equation 2.10, is used to show structures that determine optimal temporal structures that characterize and predict events. The ranking function,

\[
f(TS) = \begin{cases} 
\mu_M & \text{if } c(M) > \beta c(X) \\
\mu_M - g_{\min} \frac{c(M)}{\beta c(X)} + g_{\min} & \text{otherwise}
\end{cases}
\]

where \( \beta \) is a barrier function designed to ensure a minimum number of phase space points are within each temporal structure, \( g_{\min} \) is the minimum prediction event value, and \( c(X) \) is the cardinality of all phase space points. This particular ranking function allows the TSDM method to make predictions that have high average percent change values [3, 28].

These concepts are combined in the following TSDM method. The main goal of the TSDM method is to find temporal structure used for predicting events. The selected temporal structure is the structure with the highest fitness value found during the training period. A genetic algorithm-based optimization process, described within the TSDM method, is used to search for predictive temporal structures.

### 2.4.2 Time Series Data Mining Method

The steps for the Time Series Data Mining Method are listed below. These steps refer to diagram of the TSDM method in Figure 2.5 (shown below) and the concepts defined in Section 2.4.1. The TSDM method diagram precedes the list, and an explanation is then provided for each step in the list.
Define goal function, ranking function and optimization formulation

Select embedding dimension

Search phase space for optimal temporal structures

Embed training period time series

Observed time series

Predict events using selected temporal

Embed testing period time series

Testing Stage

Training Stage

Figure 2.5 Diagram of Time Series Data Mining Method

TSDM Method Steps

1. Define the event to be predicted.
2. Define the opportunity based on the predicted events.
3. Evaluate events with the goal function.
4. Embed training signal into a reconstructed phase space (RPS).
5. Locate temporal structures and evaluate with associated fitness values.
6. Determine predictive temporal structures in the training signal by defining a ranking function and an optimization formulation.
7. Embed testing signal into RPS.
8. Make predictions in the testing signal using the selected temporal structure.
9. Shift the window one time-step ahead and repeat steps 1-8.
The first step in applying the TSDM method to a particular problem is defining a TSDM goal. Given an observed time series, the goal is to find otherwise hidden temporal structures that are predictive of events in the time series. The events to be predicted are determined by the defined TSDM goal.

With a TSDM goal clearly defined, a given time series will be observed for predictive structures, and predictions will be made using the time series data. The TSDM method is composed of a training stage followed by a testing stage. Here, the time series, for which predictions are being made, is separated into a training signal and testing signal. The training signal is defined by a training period of $t$ weeks, starting $t$ weeks before the current time index $n$. The testing signal,

$$ Y = \{ x_n, n = B, \ldots, E \} \quad N < B < E, \quad (2.11) $$

in a time series $x_n$, is defined by the current time index $n$ from the beginning $B$, through the end of the testing signal $E$, where $N$ is the end of the training period signal. The prediction point is located at length of $t$ prediction steps away from the current time index $n$. The method makes a $t$-step prediction, denoted by $g_n = x_{n+t}$, and uses a sliding window, which slides one time-step ahead after the training and testing stages are completed for the current time index $n$. The widow size is the length of the training signal in addition to the value of the step-size $n$ for the given prediction denoted,

$$ W = \{ x_n, n = B, \ldots, E + t \} \quad N < B < E. \quad (2.12) $$

An example of the multi-step prediction process is shown in Figure 2.10. It provides both a one-step prediction and a two-step prediction. The prediction step size $t$ is chosen before experimentation.
The training stage begins by determining the TSDM objective in terms of the opportunity, event, and associated goal function $g_n$. The given time series is time delay embedded into a phase space, and an associated percent change event value is given to each time-step in the phase space. From the RPS and the associated percent change function, we form the augmented phase space. The training stage continues with the location of temporal structures in the reconstructed phase space. Temporal pattern structures are defined with the $n$ previous time series data points. After being embedded into a reconstructed phase space, each point in the phase space is a temporal pattern. The sphere surrounding that current point in the phase space with a Euclidean distance $\delta$ is a temporal pattern structure. These temporal structures are evaluated using the average of all points that lie within the temporal structure.
The TSDM method then defines an objective for determining the best temporal structure. The TSDM objective includes defining the ranking function \( f(TS) \), shown in Equation 2.8, and optimization formulation. The ranking function and the optimization are defined to determine predictive temporal patterns found in the training stage. The ranking function \( f(TS) \) rank orders the temporal structures, according to their fitness values, found during the testing stage.

The temporal structure is determined by performing a search using a simple genetic algorithm (sGA). The sGA obtains maximum fitness values by finding the parameters that maximize the ranking function \( f(TS) \), denoted \( \max_{\theta} f(TS) \). The steps in the genetic algorithm process are initialization followed by selection, fitness calculation, crossover, mutation, and reinsertion, which are performed until a stopping criterion is met. Monte Carlo search is used for random population initialization to determine the number chromosomes in the genetic algorithm. The sGA performs roulette selection, which probabilistically selects chromosomes based upon fitness value and random locus crossover, which merges chromosomes in a manner similar to sexual reproduction, to find predictive temporal structures [29-31]. The genetic algorithm evaluates fitness values and continues searching until the minimum fitness values have converged to a pre-specified convergence value. The chosen convergence value is used to halt the genetic algorithm search when the ratio of the worst fitness value to the best fitness value is equal to or above the convergence value. The results from the training stage are examined and used to make predictions in the following testing stage.

During the testing stage of the method, a \( t \)-step prediction is made. For example if \( t = 1 \), then a one-step prediction is made. The testing time series is embedded into the phase space. The selected temporal structure from the training stage is used to predict
events at the current time index. If a sequence of embedded time series points from the testing signal falls within the selected temporal structure, then a prediction is made. This predicted event is evaluated using the appropriate goal function. The results of the testing stage are evaluated, the time range window is shifted one time-step ahead, and the process repeats starting with the training stage. The entire process of training and testing continues, making predictions and evaluations for each time index \( n \), until the end of the time range \( T = \{n = 1, \ldots, N\} \). The following section presents portfolio background material used to form a basis for portfolio optimization.

2.5 Portfolio Background

A portfolio is a set of stocks from the broader market that are combined and weighted to become one investment [1, 5, 32]. Portfolios are evaluated on many criteria such as return and risk. The return of a single asset in a portfolio is the gain or loss in that asset’s value for a particular period, in percentage terms. The expected return is estimated as the average of prior returns. Portfolio returns are the combined returns of all assets in a portfolio with their associated weighting.

Risk is either the volatility of future outcomes or the probability of an adverse outcome [2, 5]. There are two types of risk. The first is unsystematic risk or company specific risk, which is unique to a company stock price time series. This type of risk can be removed through diversification, which is a technique that combines a variety of investments within a portfolio with the intention to minimize the impact of any one security on overall portfolio performance [2]. The second is systematic or market risk, which is variable risk caused by economic conditions [2]. Systematic risk cannot be minimized through diversification because it is risk that all investors and companies incur.
in the marketplace. Modern Portfolio Theory defines risk as the variance of expected returns, whereas the Capital Asset Pricing Model defines risk relative to the overall market.

Investors determine a risk vs. return trade off criterion, establishing how much risk they are willing to take. Traditionally, low risk levels are associated with low potential returns, and elevated risks levels are associated with higher potential returns. This research assumes an investor is risk adverse and will only except higher risk levels in return for a higher profit.

Diversification is a portfolio risk management technique that combines a large set of investments within a portfolio. Diversification improves risk vs. return tradeoffs by combining stocks with different risk and return characteristics from different sectors of the overall market. The cross correlation and asset allocation among these assets allows for alternate portfolios to be generated that have better risk vs. return characteristics than any one asset by itself, thus becoming the main goal for portfolio optimization. In other words, individual company risk or unsystematic risk can be minimized by properly weighting the investments in a portfolio. The next sections present the concept of portfolio optimization and the techniques used to perform it.

2.6 Portfolio Optimization

Portfolio optimization is the analysis and management of a portfolio to obtain the maximum portfolio return for a given amount of portfolio risk. Activities such as asset allocation, which divides the portfolio value among assets in the portfolio, and diversification, allow an investor to meet specific investment goals or combination of investment goals. Periodic evaluations of portfolio performance and modifications of the
weight values, also known as portfolio management, allows for various portfolio combinations to meet various optimization criteria, such as maximizing return, minimizing risk, and achieving diversification [7].

Efficient or optimal portfolios provide the greatest return for a given level of risk, or equivalently, the lowest risk for a given return given the assets in a particular portfolio [2, 5, 9, 33]. Modern Portfolio Theory provides techniques to create such efficient portfolios. The subsequent sections present Modern Portfolio Theory and the Capital Asset Pricing Model, providing explanations of risk, return, and optimal portfolios.

2.6.1 Modern Portfolio Theory

Modern Portfolio Theory (MPT), also known as mean-variance portfolio optimization, was introduced by Harry Markowitz in 1952 [9]. This theory explains how risk adverse investors can assemble portfolios that are optimal in terms of risk and expected return. Modern Portfolio Theory maintains that risk should not be viewed in an adverse context, but rather as a characteristic part of higher reward [7, 33]. Modern Portfolio Theory defines risk in terms of variance of asset returns and explains how an efficient frontier of optimal portfolios can be constructed. An efficient frontier of optimal portfolios is a set of portfolios that maximize expected return for a given level of risk or that minimize risk for a given level of return [2, 5]. The four main steps in MPT are:

- Security valuation
- Asset allocation
- Portfolio optimization
- Performance measurement.

The MPT operates under several assumptions about investor behavior [2, 5]:
1. Investors consider each investment alternative as being represented by a probability distribution of expected returns over some holding period;

2. Investors maximize one-period expected utility;

3. Investors estimate the risk of the portfolio on the basis of the variability of expected returns;

4. Investors base decisions solely on expected return and risk, so their utility curves are functions of expected return and the variance (or standard deviation) of returns only;

5. For a given level of risk, investors prefer higher returns to lower returns. Similarly, for a given level of expected return, investors prefer less risk to more risk.

These assumptions provide a basis for determining the risk and return of a portfolio, which allow for effective diversification and the ability to obtain optimal portfolios. To determine the risk of a portfolio, the expected return for each asset in a portfolio is calculated as:

\[ E(r) = \sum_{i=1}^{n} (p_i)(r_i), \]  

where \( p_i \) is the probability of the return \( r_i \) for an asset, and \( r_i \) is the geometric average rate of return for the asset. The geometric average rate of return \( GM \) for asset \( i \) is the \( n \)th root of the product of the holding period returns for \( n \) time periods denoted by

\[ GM(i) = \left[ \prod_{i=1}^{n} HPR \right]^{1/n} - 1, \]  

where \( HPR \) is the holding period return or the total return from holding an asset from beginning to end over a finite time period. The holding period return is defined as
A portfolio’s risk is the variance of the expected return of the assets in the portfolio. The variance of each asset $i$ in the portfolio is calculated as

$$
\sigma_i^2 = \sum_{j=1}^{n} [r_i - E(r_i)]^2 \ p_i,
$$

(2.16)

where $p_i$ is the probability of the possible rate of return $r_i$, and $n$ is the number of assets. This determines the risk of each asset in the portfolio. The expected return and total risk or standard deviation for the entire portfolio can then be determined. For a portfolio of $N$ assets, the total portfolio return is the weighted average of the individual returns of the securities in the portfolio

$$
r_{\text{portfolio}}(n) = \sum_{i=1}^{N} w_i r_i,
$$

(2.17)

where $w_i$ is the percent of the portfolio allocated in asset $i$, and $r_i$ is the expected rate of return for asset $i$. To calculate portfolio risk, the covariance and correlation between the assets in the portfolio is required. The covariance,

$$
\text{Cov}_{ij} = E\{[r_i - E(r_i)][r_j - E(r_j)]\},
$$

(2.18)

for two assets $i$ and $j$, is the degree in which the assets in the portfolio move together relative to their means over time [5]. Correlation is the simultaneous change in value of two numerically valued random variables. The correlation coefficient for two assets can be determined by

$$
\rho_{ij} = \frac{\text{Cov}_{ij}}{\sigma_i \sigma_j},
$$

(2.19)
where $cf$ is the correlation coefficient of returns, $\sigma_i$ is the standard deviation of $r_i$ at time index $n$, and $\sigma_j$ is the standard deviation of $r_j$ at time $n$. Using the associated weights, asset variances, resulting correlation coefficients, and covariance matrices of the assets in the portfolio, the risk of the total portfolio can be calculated. The standard deviation for a portfolio $\sigma_{\text{portfolio}}$ is

$$\sigma_{\text{portfolio}}(n) = \sqrt{\sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \text{Cov}_{ij}}, \quad (2.20)$$

where $w_i$ is the weight of asset $i$ in the portfolio, $\sigma_i^2$ is the variance of returns for assets $i$, and $\text{Cov}_{ij}$ is the covariance between returns for assets $i$ and $j$.

Alternate portfolios with various return and risk characteristics can be constructed by varying the weights of the assets in the portfolios. As stated earlier, a mean-variance efficient frontier, shown in Figure 2.11, for optimal portfolios represents the set of portfolios that has the maximum rate of return for each level of risk, or the minimum risk for every level of return [2, 5].
The efficient frontier illustrates various optimal portfolios in terms of risk and return with portfolio $H$ representing the portfolio with all the weighting in the asset with the highest return and portfolio $L$ representing the portfolio with all the weighting in the asset with the lowest risk. The other points inside the efficient curve represent portfolios that are not optimal in terms of risk vs. return.

The efficient frontier is determined through a constraint maximization process discussed in detail in [2, 5, 7, 9, 34, 35] and shown as $\max_{\text{portfolio}} r_{\text{portfolio}}(w_i, r_i)$ and $\min_{\text{portfolio}} \sigma_{\text{portfolio}}(w_i, r_i, \sigma_i)$. When the portfolio return equation is solved to obtain the maximum return of the portfolio, the portfolio risk is held constant. On the other hand, when the portfolio risk is solved to obtain the minimum risk, the portfolio return is held constant. Once equally spaced portfolios are created, portfolios are optimized to
maximize portfolio return for a given value of portfolio risk or minimize portfolio risk for a given value of portfolio return. Portfolios with equally spaced risk or return values are created to form the alternate portfolio combinations, which form the efficient frontier. For instance, when risk is held constant, \(N - 2\) equally spaced risk values between the risk of portfolios \(H\) and \(L\) are calculated. On the other hand when return is held constant, \(N - 2\) equally spaced portfolio return values between the portfolio return values of portfolios \(H\) and \(L\) are calculated.

Portfolio optimization is accomplished by iteratively adjusting portfolio asset weights. This process is repeated for each portfolio at time index \(n\) for the given time range \(T\). Modern Portfolio Theory implies that all investors should only select from portfolios that are on the efficient frontier and that investors only differ in their expectations of risk and return [5, 35]. In other words, an optimal portfolio is an efficient frontier portfolio that has the highest utility for a given investor. The following section explains the Capital Asset Pricing Model and specifically how an optimal portfolio is selected.

2.6.2 Capital Asset Pricing Model

The capital asset pricing model (CAPM) is an economic model for valuing securities by determining the relationship of risk and expected return [1, 36, 37]. The CAPM model, extending modern portfolio theory, is based on capital market theory and the idea that investors demand additional expected return for additional levels of risk [5, 36, 37]. The risk-free rate of return \(r_f\) is a theoretical interest rate returned on an investment that is completely free of risk. The 90-day Treasury bill, which is a United States government-backed security, is a close approximation, since it is virtually risk-free.
[5]. By introducing the risk-free asset, the CAPM allows for separation of risk from 
return and a model from determining the required rate of return for assets and portfolios.

The CAPM operates under several assumptions about investor behavior. The most 
important assumptions for this research are:

1. All investors are Modern Portfolio Theory investors and want portfolios that 
   are on the efficient frontier.
2. Investors can lend and borrow at the risk-free rate of return.
3. Capital Markets are in equilibrium, and all investments are properly priced 
   according to their specific risk.

The CAPM allows for further analysis of risk in assets and portfolios by introducing the 
notion of beta. Beta is a quantitative measure of the volatility of a given stock or portfolio 
relative to the overall market [1, 2, 5]. The beta $\beta_i$ value for an asset $i$ in a portfolio and 
an entire portfolio is

$$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)}, \quad (2.21)$$

$$\beta_{\text{portfolio}}(n) = \frac{Cov(r_p, r_m)}{Var(r_m)}, \quad (2.22)$$

where $r_i$ is the return for stock $i$, $r_m$ is the vector $n$-period market returns, and $r_p$ is the 
vector of $n$-period portfolio returns over the given time range. The market has a beta 
value of one. A risk-free asset has a beta value of zero. The risk-free asset has a definite 
expected return with the assumption of zero risk or zero variance of expected returns. The 
risk-free asset has zero correlation of with all other risky assets and allows for an investor 
to make alternative risk and return tradeoffs. By introducing the notion of beta and the
risk-free asset, a new efficient frontier called the Capital Market Line is derived. The Capital Market Line denoted by

$$r_{\text{portfolio}} = r_f + \beta_{\text{portfolio}}(r_m - r_f)$$

(2.23)

represents a line from the y-intercept at the risk free rate of return tangent to the original efficient frontier, shown in Figure 2.12.

![Figure 2.12 Capital Market Line](image)  

The CAPM allows for various possible combinations of investing in an efficient portfolio and the risk-free asset to be formed. This linear risk-return combination allows us to construct portfolios that are superior, in terms of risk vs. return, to portfolios on the original efficient frontier. Using of this new efficient frontier, shown in Figure 2.12, the point of tangency on the Capital Market Line is defined as the market portfolio $M$. [1,
Chapter 2  

Background

2, 5]. The market portfolio $M$ refers to a theoretical portfolio that is completely diversified containing every security available in a given market, such as stocks, bonds, options, real estate, and other forms of investments. Due to diversification, this portfolio completely eliminates unsystematic risk, encouraging investors to invest in this portfolio and borrow or lend at the risk-free rate of return. The market portfolio therefore has no unsystematic risk, which implies it only has systematic market risk or risk that cannot be diversified away. Since the market portfolio $M$ contains all available risky assets it has no unsystematic risk, and it is defined as an optimal investment choice for all investors [1, 2, 5]. Defining the market portfolio $M$, on the CML provides a suitable way to pick an optimal portfolio from any given efficient frontier.
Chapter 3 Methods

This chapter presents the combined Time Series Data Mining Portfolio Optimization method of selecting and optimizing weekly stock portfolios. It explains the adaptations of the Time Series Data Mining (TSDM) method and the modified portfolio optimization process. The TSDM method provides a predictive method for stock selection, and adaptations of Modern Portfolio Theory, and the Capital Asset Pricing Model techniques are used to optimize weekly portfolios. The chapter concludes with an overview the TSDM-Portfolio Optimization trading strategy.

Extending the Time Series Data Mining method, multiple-step weekly predictions are made and combined into weekly portfolios. Once the securities are selected by the TSDM method, they are combined into optimal weekly portfolios that maximize portfolio return for a given level of portfolio risk. Techniques used in creating optimal portfolios are adapted from Modern Portfolio Theory and the Capital Asset Pricing Model.

Combining weekly stock selection and portfolio optimization, a weekly trading strategy is created. The trading strategy buys all stocks selected from the TSDM stock selection method at the beginning of the trading week, with associated weight values determined by the associated portfolio optimization techniques. The entire portfolio is sold at the end of the trading week, and this process is repeated for each week in the given time range.

This trading strategy takes an active portfolio management approach to optimizing portfolios. An active approach is one with frequent, in this case weekly, trading activity. This is in contrast to a passive approach such as a buy and hold strategy.

The combined method, shown in Figure 3.1, performs stock selection, portfolio construction, portfolio optimization, and performance calculation. Stock price time series
data for all stocks in a given market are provided to the TSDM Stock Selection method discussed in Section 3.1. Each stock price time series is processed one at a time, and the stock selection method repeats until predictions for all stocks in the index are made. Once predictions are made, portfolios with equally weighted assets are constructed and then optimized. After portfolio optimization is complete, portfolio performance is calculated for each weekly portfolio. The following section describes the Time Series Data Mining stock selection method.

![Figure 3.1 Time Series Data Mining Portfolio Optimization Method](image)

### 3.1 Stock Selection Method

The Time Series Data Mining method is used as a stock selection tool that selects assets used in constructing weekly portfolios. The goal of TSDM stock selection, shown in Figure 3.2, is to select stocks that will increase in price. The TSDM method is extended to explore multiple time-step prediction capabilities. The multiple time-step approach to the TSDM method makes predictions out further than one time step. For instance, a one-step prediction is in the form of $g_n = x_{n+1}$, and a $t$-step prediction is in the form of $g_n = x_{n+t}$, where $t$ is the number of weeks ahead the prediction is being made.
Following the steps listed in Section 2.4.2, weekly stock predictions are made with different prediction-step lengths. Each signal is embedded into a RPS. Temporal patterns are defined using the previous three points of stock closing price data, with an embedding dimension of 3. Events are triggered and opportunities are created if a temporal pattern in the predictive testing stage is within a region defined by the optimal temporal structure found in the training stage. The associated percent change function shown,

\[ g_n = \frac{x_{n+t+1} - x_{n+t}}{x_{n+t}}, \quad (3.1) \]

allows for a value to be given to a multi-step prediction made during the testing stage of the Time Series Data Mining Method. This function links temporal structures, found in the training stage, with event that occur in the future, such as the desired increase in stock price.

Once stock selections are made, predictions are combined into weekly portfolios. A dynamic portfolio matrix, \( p \) stocks by \( N \) weekly time periods, is constructed. Portfolio
assets and performance will be different for each weekly portfolio due to the active portfolio management approach in which each portfolio is bought at the beginning of the week and sold at the end of the week. Weekly stock predictions are made with associated goal function values (weekly stock price percent change) that are either positive, negative, or zero. A positive prediction value means that the stock price increased. A negative prediction value means the stock price decreased for that week. A prediction value of zero means no prediction was made in that week for that stock. The next section describes the modified portfolio optimization process.

3.2 Modified Portfolio Optimization Method

Once the equal weighted portfolios are created, portfolio optimization techniques are used to optimize the weekly portfolios. The goal of the portfolio optimization is to maximize portfolio return for a given level of portfolio risk. As stated earlier, the risk of a portfolio is defined as the standard deviation of expected returns of the assets in the portfolio. Modern Portfolio Theory (MPT) provides techniques to adjust the portfolio weights, to maximize portfolio return for each level of risk.

The efficient frontier of portfolios represents the set of optimal portfolios from which to choose. To construct an efficient frontier from a given equally weighted portfolio, a weight adjustment process must be conducted. The weight adjustment process is a constrained optimization problem formulated by maximizing portfolio return for given portfolio risk values. The predictive process uses expected returns generated from the training period signal to create the covariance matrix of expected returns used in calculating portfolio risk. The covariance matrix represents the variance between
expected returns for each asset in the portfolio. The method iteratively adjusts weight values using the process listed below:

1. Calculate portfolio return $r_{\text{portfolio}}(t)$, $t = 1$ and portfolio risk $\sigma_{\text{portfolio}}(t)$, $t = 1$ with equally weighted assets.

2. Hold portfolio risk, $\sigma_{\text{portfolio}}(t)$, constant and adjust weights $(w_1, \ldots, w_n)$, to achieve a higher portfolio return $r_{\text{portfolio}}(t)$.

3. If $\sum (w_1, \ldots, w_n) = 1,$ $w_n \neq 0,$ $\forall n$ and $r_{\text{portfolio}}(t) > r_{\text{portfolio}}(t-1),$ $t = (2, \ldots, n)$

Continue to adjust portfolio weights to achieve a higher return value.

4. If $r_{\text{portfolio}}(t) \leq r_{\text{portfolio}}(t-1)$ then $r_{\text{portfolio}} = \max(r_{\text{portfolio}}(t), r_{\text{portfolio}}(t-1))$.

Using the Capital Asset Pricing Model (CAPM), portfolio returns and risk can be separated using the Capital Market Line (CML) equation. Making assumptions about the risk-free rate of return, a line can be extended from the risk-free rate of return, tangent to the efficient frontier, constructing the CML. Incorporating previously stated assumptions of the CAPM, and the derived market portfolio $M$ from the CML, a new portfolio $M'$ is now introduced. The new portfolio $M'$ is the tangent point located on the constructed CML extended from the risk-free rate of return. The portfolio $M'$, shown in Figure 3.3, is the equivalent, in terms of location on the CML, to the original optimal market portfolio $M$, but using only the stocks selected during the Time Series Data Mining stock selection process. This concept is essential, because it provides the criterion for selecting an optimal portfolio from the new efficient frontier created by the CML. An optimal portfolio, for performance calculation purposes, is defined as the optimal market portfolio $M'$ or the portfolio with the lowest risk if the optimal market portfolio $M'$ return is less
than the risk-free rate of return $r_f$. The next section presents and explains the complete Time Series Data Mining Portfolio Optimization Method.

**Figure 3.3 Model Market Portfolio $M'$**

### 3.3 **Time Series Data Mining Portfolio Optimization Trading Strategy**

The combined trading strategy involves using the TSDM Stock Selection method, discussed in Section 3.1 to make weekly buy or do nothing signals for each stock in the given stock market index. These predictions are made for multiple weekly time steps ahead. This prediction process takes the weekly stock time series data and makes a $t$-step prediction using repeated experiment runs with different prediction time-step parameters. After the initial stock selection process, the weekly portfolios are constructed and
optimized using the adapted portfolio optimization techniques discussed in Sections 2.6 and 3.2.

The steps to formulate a trading strategy using the Time Series Data Mining Portfolio Optimization approach are listed below:

1. Determine entire time range for stock predictions including training period signal length.
   a. Determine portfolio strategy time period (daily, weekly, monthly, etc.).

2. Determine desired prediction step size \( t \geq 1 \).

   a. Choose stock market index for stock price data set.
   b. Define goal function and ranking function.
   c. Determine time series embedding and temporal pattern length.
   d. Define genetic algorithms parameters (Section 2.4.2).

   a. \( p \) stocks by \( N \) time periods.

5. Perform Mean Variance Portfolio Optimization (Sections 2.6 and 3.2).
   a. Create efficient frontier.
   b. Select an approximate risk-free rate of return.
   c. Create Capital Market Line.
   d. Select optimal portfolio.
   e. Repeat for a. through d. for all weekly portfolios

Portfolio performance analysis is performed on all generated portfolios for that time range. Weekly portfolio return performance and total performance are measured and compared against the overall market performance as a baseline. The TSDM Stock Selection method prediction accuracy results are compared against the market baseline prediction accuracy measure. Portfolio performance analysis and results, including return, risk, transaction cost, prediction accuracy, and Sharpe’s ratio are presented in Chapter 4.
Chapter 4 Evaluation

This chapter presents an evaluation of the combined method discussed in Chapter 3. Historical stock market data is used to evaluate the Time Series Data Mining Portfolio Optimization (TSDMPO) trading strategy. The data is comprised of stock price time series of stocks in a particular market index. The chapter contains an explanation of the TSDMPO trading method stock market application, experimental set-up, and experimental results including a transaction cost model.

4.1 Stock Market Application

The combined Time Series Data Mining Portfolio Optimization method is used to identify profitable trading opportunities and create wealth in an active trading environment. The evaluation of this trading strategy is performed in a simulated market where stocks are bought on the first trading day of the week and sold on the last trading day of the week. In applying any trading strategy in an actual market setting, investors must pay transaction costs in order to trade securities. The weekly trading strategy makes weekly predictions over specific time ranges and combines the predictions into weekly portfolios used to increase profit, outperform market return benchmarks, and overcome transaction costs.

To simulate an active trading environment, investors are able to implement this trading strategy using large online trading sites such as TDWaterhouse.com, Etrade.com, and Ameritrade.com, which have allowed various types of investors to set up and easily manage their own investments. Combining the availability of current financial data access and the ability to independently manage investments the Time Series Data Mining Portfolio Optimization method trading strategy is explored in a simulated market.
environment where transaction cost are taken into consideration. The investment strategy takes advantage of predictive stock selection and optimal asset allocation to trade portfolios weekly. The next section describes the transaction cost model used to determine simulated model portfolio returns.

4.2 Transaction Cost Model

A transaction is the buying or selling of a security, and transaction costs are those associated with trading securities [27,36]. When making trades in the stock market, investors incur transaction costs that are paid for each transaction made. Transaction cost can erode the total returns gained from investments. These adverse effects must be considered to determine whether the trading strategy is able not only to increase wealth but also overcome the associated cost with making those trades. Transaction costs have two components. One cost is broker commissions or fees that are charges assessed by an agent in return for arranging the purchase or sale of a security [33-35]. Another cost is the spread, commonly referred to as bid-ask spread, which is the difference between the ask price (the price at which an investor is willing to sell a particular security in the secondary market) and the bid price (the price at which an investor is willing to buy a particular security in the secondary market) [33-35].

The commission value is determined by the typical price paid to buy or sell shares of stock at an online trading site such as TDWaterhouse.com or Etrade.com. The commission per transaction is $10 per stock or $20 for a round trip (buy and sell). A model for the approximate bid-ask spread was developed by Roll [36]. Assuming that the markets are efficient and that the probability distribution of observed price changes is stationary in short intervals, the spread is modeled by the first order covariance of
successive price changes [36]. A modified version of the equation is used to model bid-ask spread and neglects the downward bias in the original equation, which makes spread values negative [36]. The bid-ask spread is modeled as:

\[ BAS_n = 2\sqrt{\text{cov}(S(x_n))}, \]  

(4.1)

where \( BAS_n \) is the bid-ask spread at time index \( n \), and \( S(x_n) \) is the stock closing price at time \( n \). The total transaction cost for a security at time index \( t \) is the sum of commissions plus the bid-ask spread shown,

\[ TC_n = BAS_n + C_n, \]  

(4.2)

where \( TC_n \) is the transaction cost at time index \( n \), \( BAS_n \) is the bid-ask spread at time index \( n \), and \( C_n \) is the commission at time index \( n \). The following section explains the experiment set-up used in evaluating the TSDMPO trading strategy.

### 4.3 Experiments

The experiments are divided into groups based on the prediction time-step for each experiment. Prediction time steps 1, 2, 3, and 4 are used in exploring the multi-step capabilities of the Time Series Data Mining Stock Selection method. Experiments are also grouped based on the chosen model testing time range of the chosen data set.

The data set used in the experiments is obtained from http://finance.yahoo.com and is comprised of weekly stock market data from the Dow Jones Industrial Average. The Dow Jones Industrial Average (DJIA) is a price-weighted average of thirty large capital stocks traded on the New York Stock Exchange. The stock listing for the Dow Jones Industrial Average is shown in Appendix A.1. The stock market data is in the form of weekly open price, close price, high price, low price, and trading volume. The data set...
spans from January 1, 2002 to January 1, 2003 and January 1, 2003 to January 1, 2004. Theses time ranges were selected to test the method in both bull (generally rising stock prices) and bear (generally declining stock prices) market conditions.

Time Series Data Mining Stock Selection parameters involving the training period and genetic algorithm based optimization are held constant through each experiment. The time series is embedded with a dimension of 3, which creates predictive structures using the three weeks of closing price data including the current time point and the two previous points. The initial genetic algorithm population is set to 30 to create a population large enough for the genetic algorithm to make subsequent solution generations. The algorithm has halting criterion set to stop the genetic algorithm search when fitness values converge to a value set at 0.9 multiplied by the maximum fitness value. The training period is 26 weeks and was chosen by empirically comparing results of each market index experiment using training ranges varying form 5 weeks to 52 weeks.

Portfolio optimization parameters include the initial portfolio value and the risk-free rate of return used in calculating model returns and determining optimal portfolio selection. The initial portfolio value is reset to $100,000 dollars at the beginning of every trading week to provide a basis on calculating weekly returns and adjustments for transaction cost. The risk-free rate of return is the 90-day Treasury bill rate of return at the beginning of the time range. The 90-day Treasury bill rate of return was 1.68 % at January 1, 2002 and 1.19 % at Jan 1, 2003.
4.4 Results

This section presents results for weekly-optimized portfolios from the Dow Jones Industrial Average data used in experiments. Optimized portfolios or mean variance efficient portfolios are defined Chapter 2 and further discussed Chapter 3. These portfolios are described by the market portfolio $M'$ defined by the Capital Market Line defined in Chapter 3. Results from the prediction-step experiments conducted in a stock market index will be compared to the same complete market index as a benchmark. The model return results from optimized portfolios generated from stock selection using the Dow Jones Industrial Average index will be compared against the DJIA index market rate of return and buy and hold returns. The model risk will be compared using portfolio beta values, which measure risk relative to the overall market. The index benchmarks are performance for the entire index, while the model portfolios are specific segments of the market.

4.4.1 Portfolio Return

Portfolio returns are presented in this section and are defined in Chapters 2 and 3. Portfolio returns are calculated by using the associated weight values determined from the portfolio optimization process and described in Sections 3.2 and 3.3. A vector of optimal portfolio returns,

$$r_{optimal} = [r(n)_{portfolio}, n = 1, ..., N]$$

contains the optimal portfolio returns for each week over the entire time range $T$. The combined model rate of return is the geometric average of weekly portfolio returns over the entire time range $T$ shown as
The adjusted model rate of return is the combined model rate of return with the average weekly transaction cost, shown in Section 4.2, subtracted from it denoted by

\[ rr_{\text{Model}}^*(n) = rr_{\text{Model}}(n) - \left( \frac{\sum_{n=1}^{N} TC(n)}{N} \right). \]  

(4.5)

The total model return is the product of the adjusted model rate of return for the time range,

\[ R_{\text{Model}} = \left( \prod_{n=1}^{N} rr_{\text{Model}}(n) \right) - 1. \]  

(4.6)

Weekly portfolio return values are calculated and then averaged to obtain an overall performance value for the time range. The average model risk is the mean of all weekly portfolio risk values over the time range. Tables 4.1 and 4.2 provide numerical results for the Time Series Data Mining Portfolio Optimization model, using Dow Jones Industrial Average stock data, with prediction steps 1, 2, 3, and 4.

<table>
<thead>
<tr>
<th>Dow Jones Industrial Average</th>
<th>Prediction Time-Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/01/2002 – 1/01/2003</td>
<td>1</td>
</tr>
<tr>
<td>Model Rate of Return</td>
<td>1.898 %</td>
</tr>
<tr>
<td>Adjusted Model Rate of Return</td>
<td>1.897 %</td>
</tr>
<tr>
<td>Average Weekly Transaction Cost ($)</td>
<td>97.00</td>
</tr>
<tr>
<td>Total Model Return</td>
<td>151.240 %</td>
</tr>
</tbody>
</table>

Table 4.1 Dow Jones Industrial Average Return Performance 1/01/2002 – 1/01/2003
The model rate of return was greater than the average market rate of return in the January 1, 2002 through January 1, 2003 time period. The average weekly market rate of return was -0.315% for the time period. This also led to the total model return also being greater than the market total buy and hold return in the time period. The total market buy and hold return was –18.070% for this time period. Average weekly transaction cost had little effect on overall return performance due to the small number of stocks selected on a weekly basis and low estimated bid ask spread values.

<table>
<thead>
<tr>
<th>Dow Jones Industrial Average 1/01/2003 – 1/01/2004</th>
<th>Prediction Time-Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Rate of Return</td>
<td>1  2  3  4</td>
</tr>
<tr>
<td>2.249 %</td>
<td>1.876 % 1.704 % 2.302 %</td>
</tr>
<tr>
<td>Adjusted Model Rate of Return</td>
<td>2.248 % 1.875 % 1.703 % 2.301 %</td>
</tr>
<tr>
<td>Average Weekly Transaction Cost($)</td>
<td>104.00 104.00 112.00 106.00</td>
</tr>
<tr>
<td>Total Model Return</td>
<td>197.324 % 144.056 % 121.256 % 178.456 %</td>
</tr>
</tbody>
</table>

Table 4.2 Dow Jones Industrial Average Return Performance 1/01/2003 – 1/01/2004

The model rate of return was greater than the average market rate of return in the January 1, 2003 through January 1, 2004 time period. The average weekly market rate of return was 0.351% for the time period. This also led to the total model return also being greater than the market total buy and hold return in the time period. The total market buy and hold return was 24.333% for this time period. Average weekly transaction cost were higher than the previous year due to the model making more selections in better bull market conditions. However, the transaction cost still had little effect on overall return performance.

The poor market conditions that existed from January 1, 2002 through January 1, 2003 led to an overall lower performance compared to the results for the time period
spanning from January 1, 2003 through January 1, 2004. In both experiments, the multiple step predictions were able to provide positive results by outperforming the overall market and overcoming associated transaction costs. The overall portfolio performance results were consistent between both experiment time ranges, by showing a slight decrease in the rate of return as the prediction step increased, except for the increase in rate of return for prediction step size 2 in year 2002 experiments and the increase for rate of return in prediction step size of 4 in year 2003 experiments.

Figures 4.1 through 4.4 show the 1, 2, 3, and 4 step cumulative weekly returns for the TSDMPO model vs. the benchmark for the year 2002. Figures 4.5 through 4.8 show the 1, 2, 3, and 4 step cumulative weekly returns for the model vs. the benchmark for the year 2003. The graphical representation of these results shows how the model compares to the entire market, as a benchmark, over the given time ranges.
Figure 4.1 represents the one-step TSDMPO model cumulative weekly rate of return compared with the DJIA index cumulative weekly rate of return. This plot shows that the TSDMPO model outperforms the benchmark index over the time period. The model rate of return is lower than the benchmark for a very brief period of weeks early in the time period and performs very strongly later in the time period. The one-step prediction model shows greater rate of return volatility than the market as show by Figure 4.1 with larger moves in cumulative weekly return. The model’s largest one week loss was 18.08% during week 26, and the model’s largest one week gain was 26.63% during week 40, for this time period.
Figure 4.2 represents the two-step TSDMPO model cumulative weekly rate of return compared with the DJIA index weekly rate of return. This plot shows that the TSDMPO model outperforms the benchmark index over the time period. The total model rate of return never goes lower than the rate of return benchmark index and has strong performance in the second half of the year. The two-step prediction model shows greater rate of return volatility than the market as show by Figure 4.2, but does not show as much volatility as the one-step prediction model. The model’s largest one week loss was 13.56% during week 46, and the model’s largest one week gain was 26.07% during week 39, for this time period.
Figure 4.3 represents the three-step TSDMPO model cumulative weekly rate of return compared with the DJIA index weekly rate of return. This plot shows that the TSDMPO model outperforms the benchmark index over the time period. The total model rate of return has periods of under performance in the first half of the time period, but performs stronger later in the time period by outperforming the market rate of return benchmark. The three-step prediction model shows greater rate of return volatility than the market as show by Figure 4.3, but shows less volatility than the one-step and two-step prediction models. The model’s largest one week loss was 16.92% during week 31, and the model’s largest one week gain was 12.80% during week 26, for this time period.
Figure 4.4 represents the four-step TSDMPO model cumulative weekly rate of return compared against the DJIA index weekly rate of return. This plot shows that the TSDMPO model outperforms the benchmark index over the time range. The model rate of return has periods where performance is relatively close to the return benchmark index in the first half of the time range, but shows stronger performance later in the time range. The four-step prediction model shows greater rate of return volatility than the market and similar rate of return volatility to the three-step model, but shows less volatility than the one-step and two-step prediction models. The model’s largest one week loss was 11.54% during week 19, and the model’s largest one week gain was 12.76% during week 25, for this time period.
Figure 4.5 represents the one-step TSDMPO model cumulative weekly rate of return compared against the DJIA index weekly rate of return. This plot shows that the TSDMPO model outperforms the benchmark index over the time range. The model rate of return has periods where performance is lower that the rate of return benchmark index early in the time range, but outperforms the benchmark significantly later in the time range. The one-step prediction model shows greater rate of return volatility than the market, but the majority of the volatility is in the positive direction. The model’s largest one week loss was 6.35% during week 1, and the model’s largest one week gain was 14.90% during week 38, for this time period.
Figure 4.6 represents the one-step TSDMPO model cumulative weekly rate of return compared against the DJIA index weekly rate of return. This plot shows that the TSDMPO model outperforms the benchmark index over the time range. The model rate of return has periods where performance is slightly lower than the rate of return benchmark index early in the time range, but then quickly outperforms the benchmark throughout the time range. The two-step prediction model shows greater rate of return volatility than the market and has similar volatility to the one-step prediction model. The model’s largest one week loss was 8.30% during week 10, and the model’s largest one week gain was 12.67% during week 47, for this time period.
Figure 4.7 represents the one-step TSDMPO model cumulative weekly rate of return compared against the DJIA index weekly rate of return. This plot shows that the TSDMPO model outperforms the benchmark index over the time range. The model rate of return has periods where performance varies around the rate of return benchmark index early in the time range, but strongly outperforms the benchmark later in the time range. The three-step prediction model shows greater rate of return volatility than the market, but is less volatile than the one-step and two-step prediction models. The model’s largest one week loss was 10.88% during week 42, and the model’s largest one week gain was 10.62% during week 44, for this time period.
Figure 4.8 represents the one-step TSDMPO model cumulative weekly rate of return compared against the DJIA index weekly rate of return. This plot shows that the TSDMPO model outperforms the benchmark index over the time range. The model rate of return has a brief period where performance is lower than the rate of return benchmark index early in the time range, but significantly outperforms the benchmark later in the time range. The four-step prediction model shows greater rate of return volatility than the market and has similar volatility to the one-step and two-step prediction models. The model’s largest one week loss was 8.84% during week 8, and the model’s largest one week gain was 18.61% during week 16, for this time period.

Figures 4.1 though 4.8 plot the model return adjusted for transaction cost at different prediction time steps within the two time periods. The results from each
experiment are consistent with numerical results and show the model outperforming the market benchmark as time continues. From January 1, 2002 through January 1, 2003, the return performance was positive despite negative market conditions. From January 1, 2003 through January 1, 2004, the return performance improved over the previous year’s performance in three out of four prediction time steps.

Figure 4.9 shows an example of an observed optimal portfolio that occurred in 2003 using the TSDMPO trading strategy with a one-step prediction. This visual representation shows the selection of an optimal portfolio, which is theoretically explained in Section 2.6. The Optimal Risk Portfolio $M'$ represents the optimal portfolio selected during experimentation. The graph also shows the Capital Market Line extending from the risk-free rate of return, $r_f$ at 1.19%. The next section evaluates portfolio risk and overall model risk performance. The next section presents model portfolio risk results.
An evaluation of model portfolio risk is presented in this section. The risk of a portfolio can be determined in various ways. Traditional modern portfolio theory determines risk, a posteriori, as the variance and standard deviation of expected returns. The Capital Asset Pricing Model determines risk in terms of beta, $\beta$. Portfolio risk, $\sigma_{\text{portfolio}}$, and portfolio beta, $\beta_{\text{portfolio}}$, are defined in Chapters 2 and 3. Other popular measures of risk have emerged and will allow for further risk analysis of the weekly portfolios created during the Time Series Data Mining Portfolio Optimization method. The Sharpe’s Ratio is calculated to further analyze the risk vs. return of generated weekly portfolios.

Figure 4.9 Observed Optimal Portfolio w/Efficient Frontier

**4.4.2 Portfolio Risk**

*Optimal Capital Allocation*

<table>
<thead>
<tr>
<th>Risk (Standard Deviation)</th>
<th>Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0115</td>
<td>$r_f$</td>
</tr>
<tr>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td>0.0125</td>
<td></td>
</tr>
<tr>
<td>0.013</td>
<td></td>
</tr>
<tr>
<td>0.0135</td>
<td></td>
</tr>
<tr>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td>0.0145</td>
<td></td>
</tr>
</tbody>
</table>

$M'$
The Sharpe’s ratio is a ratio developed by William Sharpe in 1966 to measure risk-adjusted performance [37]. The risk adjusted return measures how much risk a portfolio assumes to earn its returns. This is usually expressed as a number or a rating. The Sharpe’s ratio $sr$ is defined as,

$$sr = \frac{r_{portfolio} - r_f}{\sigma_{portfolio}}$$  

(4.7)

where $r_{portfolio}$ is the model average portfolio return, $r_f$ is the market rate of return, and $\sigma_{portfolio}$ is the average portfolio standard deviation for the time range. The Sharpe ratio determines whether the returns of a portfolio are because of wise investment decisions or a result of taking excess risk. This ratio is useful in comparing portfolios and assets in terms of volatility to return. A high Sharpe’s Ratio implies the portfolio or stock is realizing sufficient or good returns for each unit of risk. The measure is used here to evaluate the model average portfolio return and risk results.

Tables 4.3 and 4.4 contain model portfolio risk results. Weekly portfolio risk values are calculated for each risk measure. These weekly values are averaged to obtain an overall performance value for the time range. The average portfolio risk is the mean of all weekly portfolio risk values $\sigma_{portfolio}$ over the time range. The average portfolio beta is the mean of all beta values $\beta_{portfolio}$ over the time range. The Shape’s Ratio is calculated using adjusted model rate of return, risk-free rate of return and, average model risk.
Table 4.3 Dow Jones Industrial Average Risk Analysis 1/01/2002 – 1/01/2003

<table>
<thead>
<tr>
<th>DJIA</th>
<th>Prediction Time-Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/01/2002 – 1/01/2003</td>
<td>1</td>
</tr>
<tr>
<td><strong>Average Portfolio Risk</strong></td>
<td>3.526</td>
</tr>
<tr>
<td><strong>Average Portfolio Beta</strong></td>
<td>2.534</td>
</tr>
<tr>
<td><strong>Model Sharpe’s Ratio</strong></td>
<td>3.973</td>
</tr>
</tbody>
</table>

Table 4.4 Dow Jones Industrial Average Risk Analysis 1/01/2003 – 1/01/2004

<table>
<thead>
<tr>
<th>DJIA</th>
<th>Prediction Time-Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/01/2003 – 1/01/2004</td>
<td>1</td>
</tr>
<tr>
<td><strong>Average Portfolio Risk</strong></td>
<td>2.608</td>
</tr>
<tr>
<td><strong>Average Portfolio Beta</strong></td>
<td>2.051</td>
</tr>
<tr>
<td><strong>Model Sharpe’s Ratio</strong></td>
<td>6.152</td>
</tr>
</tbody>
</table>

The average portfolio risk values for each prediction step are similar to each other, with an overall model average of 3.22 in 2002 experiments and 2.74 in 2003 experiments. The average portfolio betas for each prediction step are also similar to each other, with an overall model average of 2.58 in the 2002 experiments and 2.10 in 2003 experiments. The Sharpe’s ratio value reported is the average value over the time range, and results do not assume that model portfolio returns must exceed the risk-free rate of return given by the 90-day Treasury bill at the start of each period, with 1.68 % on January 1, 2002 and 1.19 % on January 1, 2003. The average Sharpe’ ratio was 3.28 for year 2002 experiments and 5.32 for year 2003 experiments. Sharpe’s ratio values greater than one are good, and values greater than two are outstanding. This implies that
experimental results were better than outstanding [43]. The next section shows portfolio prediction accuracy results.

4.4.3 Prediction Accuracy

During the testing stage of the Time Series Data Mining (TSDM) method, predictions are evaluated. Stock predictions are made and then given values that are either positive, negative, or zero. A positive prediction value means that stock price went up, and a negative prediction value means the stock price went down for the week. The underlying TSDM trading objective is to find patterns that are predictive of increases in a stock price. Determining prediction accuracy of the stock selection process is another indicator of how much risk is taken in investing in these portfolios. If the stock selection tool is making accurate predictions for positive trading opportunities, then there is less inherent risk in making trades based on the model. The ratio between the number positive trades and the number of negative trades is an important measure to determine whether the stock selection process is accurate given any market conditions. If market conditions are good, the stock selection should be able to select more stocks with positive gains to add to a weekly portfolio. In contrast, if market conditions are poor, the model may select fewer if any stocks to add to the weekly portfolios.

The prediction accuracy also plays a role in the risk of each portfolio. If the predictions are more accurate, there is less unsystematic risk present in each weekly portfolio. In comparison to the market baseline positive prediction accuracy, if the model prediction accuracy is higher, there is less risk than the market in our portfolios.
The prediction accuracy determines the total number of trades in a given time index \( n \) and gives a percent value equal to the ratio between the number of positive trades and the total number of trades made for that week. The prediction accuracy is defined as

\[
P_n = \frac{PT_n}{TN_n},
\]  

(4.8)

where \( PT_n \) is the number of positive trades, and \( TN_n \) is the total number of trades in the current week \( n \). The total prediction accuracy is an average of the weekly prediction accuracies over the entire time range \( T \) denoted by

\[
P_{avg} = \frac{\sum_{n=1}^{N} P_n}{N}.
\]  

(4.9)

The model percent accuracy is measured against a complete market baseline percent accuracy. The market baseline percent accuracy measure incorporates all possible weekly trades for the time period and determines the number of weekly trades that had positive percent gains and negative percent gains. This comparison gives a baseline against which to measure how well our prediction accuracy fares against purchasing all given assets in a market index. Tables 4.5 and 4.6 include results on the model and market accuracy.

<table>
<thead>
<tr>
<th>DJIA</th>
<th>Prediction Time-Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/01/2002 – 1/01/2003</td>
<td>1</td>
</tr>
<tr>
<td><strong>Positive Model Accuracy</strong></td>
<td>0.500 %</td>
</tr>
<tr>
<td><strong>Positive Market Baseline</strong></td>
<td>0.452 %</td>
</tr>
</tbody>
</table>

Table 4.5 Prediction Accuracy Analysis 1/01/2002 – 1/01/2003
The prediction accuracy for the model was greater than the market baseline accuracy in the one-step prediction, with date range January 1, 2002 through January 1, 2003, experiment. In addition, this was the only experiment out of the four prediction model experiments with date range, January 1, 2002 through January 1, 2003, that had prediction accuracy greater than the market baseline accuracy. In contrast, three out of the four prediction model experiments with date range, January 1, 2003 through January 1, 2004, had prediction accuracies greater than the market baseline accuracy. The next chapter concludes the thesis, providing insights into the work and future research directions.

<table>
<thead>
<tr>
<th>DJIA 1/01/2003 – 1/01/2004</th>
<th>Prediction Time-Step</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Positive Model Accuracy</td>
<td>0.594%</td>
</tr>
<tr>
<td>Positive Market Baseline</td>
<td>0.537%</td>
</tr>
</tbody>
</table>

Table 4.6 Prediction Accuracy Analysis 1/01/2003 – 1/01/2004
Chapter 5 Conclusions and Future Work

This thesis presents a profitable stock trading strategy by combining a temporal data mining based stock selection approach with portfolio optimization techniques. The background information for these techniques are found in Chapter 2. The combined method and trading strategy details are discussed in Chapter 3. Research conclusions include comparisons and discussions of results, presented in Chapter 4, to provide an insight and a summary of the research. Future work recommendations, to continue progress made by this thesis, are provided at the end of this chapter.

5.1 Research Conclusions

The combined Time Series Data Mining Portfolio Optimization model was able to overcome transaction cost and outperform the market benchmark returns in all prediction steps for two time periods spanning from January 1, 2002 to January 1, 2003 and from January 1, 2003 to January 1, 2004. The prediction model is capable of looking further ahead and making predictions that are further out than one-step and achieve desired results. This predictive ability allows investors to make longer-term decisions and possibly avoid additional transaction cost due to fewer transactions. The success of the multi-step approach shows the stock selection model’s predictive ability to select stocks with positive returns over various prediction horizons.

In further evaluating the TSDM stock selection method, model prediction accuracies also demonstrated that the stock selection method has predictive ability. Prediction accuracy results show that the model prediction accuracy was lower in year 2002 experiments than in year 2003 experiments. The model had higher prediction accuracy than the market baseline in four out of eight total experiments, with three of
those accuracy measurements coming in year 2003 experiments, which experienced favorable market conditions. The stock selection model has the capability to work better in good market conditions, while still achieving desired return performance in both good and bad overall market conditions.

When considering prediction accuracy as a part of portfolio risk, due to an investor’s ability to make positive or negative portfolio stock selections, the model assumed more risk in the year 2002 experiments than it did in the year 2003 experiments, due to the lower accuracy levels. Year 2002 experiments averaged 43.5% accuracy and year 2003 averaged 55.7% accuracy. The year 2002 had overall bad market conditions. These market conditions forced the combined model to take on more portfolio risk to achieve positive returns in Chapter 4. The model also took on more risk, in a traditional sense, shown by the portfolio beta values. The model average beta risk values were at least two times greater (2.58 and 2.10) than the market beta, which has a beta value of 1. The model beta values show that generated portfolios on average have a higher risk level than the overall market. This risk is mainly due to a lack of diversification in weekly portfolios. The generated model portfolios have fewer stocks in them than the number of stocks in the overall benchmark index.

The returns of the initial stock portfolios are improved using the portfolio optimization techniques discussed in Section 3.2. The process of rebalancing the portfolio weights to achieve better risk vs. return characteristics also contributes to the overall model return. The optimal portfolios constructed have better risk vs. return characteristics than portfolios that are not optimized with the same set of assets. This has been demonstrated both in theory and now in practice using the Time Series Data Mining
Portfolio Optimization trading strategy. As stated in Chapter 4, returns are calculated based on a $100,000 portfolio value with an adjustment for the transaction cost associated with the weekly trading strategy. Transaction cost will have more of an effect on the overall portfolio return if the initial weekly portfolio value is lower than $100,000 and less of an effect if the initial weekly portfolio value is higher than $100,000. Due to higher portfolio values, more shares of each stock can be purchased and added to a portfolio producing higher overall returns.

The combined TSDMPO model achieved all previously stated goals including outperforming the overall market in bull (good) and bear (bad) market conditions. This trading strategy can now be used to trade in an actual market setting using all widely available and frequently traded stocks with sufficient data resources. Using the TSDMPO trading strategy on a weekly basis helps overcome transaction cost associated with trading and allows an investor to realize profits over a given time range. The results presented here are for a one-year time period, but could be extended for longer time periods to obtain similar results due to the adaptive nature of the stock selection method. The next section discusses future work to continue the research presented in this thesis.

5.2 Future Work

Future work lies in the area of extending the predictive capabilities of the Time Series Data Mining stock selection method, incorporating short selling strategies, and investigating options data sets. The TSDM stock selection method could be extended to use multiple predictive structures with various lengths. These multiple predictive structures with various lengths will require higher embedding dimensions to capture temporal patterns in the reconstructed phase space.
The use of options data should also be considered in trying to make predictions and developing trading strategies. Using the actual options price data or the underlying option strike price data are two possible data sets that could be used to make stock selections from. Also return performance could be calculated using either the underlying stock price of the option or the actual option prices.

The current work focuses on a long trading strategy. Trading long is exactly like the trading performed in this thesis, in which a stock is bought and held to be sold later at a higher price. Short selling is the selling of a security that an investor does not currently own, and the transaction is completed by the purchase and delivery of a security borrowed by the seller [5]. A short selling strategy profits from being able to buy the stock at a lower price than the price at which they sold short. Incorporating a short selling strategy should be investigated to see if changing the model objective can achieve results that are similar those reported in this research.


Practice of Knowledge Discovery in Databases (PKDD'01), Freiburg, Germany, 2001.


Appendix

A.1 Dow Jones 30 Market Index Stock Listings (as of 1/01/2004)

Alcoa - AA
American Express - AXP
AT&T - T
Boeing - BA
Caterpillar - CAT
Coca-Cola - KO
Citigroup - C
Disney - DIS
DuPont - DD
Eastman Kodak - EK
Exxon Mobil - XOM
General Electric - GE
General Motors - GM
Hewlett-Packard - HWP
Home Depot - HD
Honeywell - HON
IBM - IBM
Intel - INTC
International Paper - IP
Johnson & Johnson - JNJ
McDonald's - MCD
Merck - MRK
Microsoft - MSFT
3M - MMM
JP Morgan - JPM
Philip Morris - MO
Proctor & Gamble - PG
SBC Communications - SBC
United Tech - UTX
Wal-Mart - WMT